



THE IRIS CENTER

CASE STUDY UNIT



ALGEBRA (PART 1)

APPLYING LEARNING STRATEGIES TO BEGINNING ALGEBRA

Created by

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For an Instructor’s Guide to this case study unit, please email your full name, title, and institutional affiliation to the IRIS Center at iris@vanderbilt.edu.

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ALGEBRA INTRODUCTION

GENERAL GUIDELINES FOR TEACHING ALGEBRA

Many students, including those with disabilities, have difficulty with the abstract concepts taught in algebra. The following guidelines should be considered when teaching algebra:

- Curriculum – Often times, the curriculum adopted by school districts moves too quickly and students may not fully understand the current concept before they are forced to move on to the next concept. It is important that teachers provide students with ample time to learn the concept and a sufficient number of opportunities to practice the concept.
- Language of mathematics – It is important that students be able to define and use algebra terminology. A list of important vocabulary terms and strategies for teaching them are included in this case study.
- Prerequisite skills – Students must master prerequisite skills prior to learning algebra. These skills include but are not limited to basic facts, problem solving skills, and probability skills. It may be necessary to review these skills prior to working with algebra concepts.
- Modeling by teacher – Teachers must model strategies prior to allowing students to complete work on their own. During modeling, teachers talk aloud as they demonstrate how to solve a problem. This should be continued until enough problems have been modeled so that students understand the concept and how to use the manipulatives provided.
- Real-life examples – It is imperative that algebra problems be related to real-life situations. Students often ask why algebra is necessary; relating it to real-life situations will encourage the connection. It is also important for students to have strategies for deciding how to set up the problems they need to solve.
- Effective instruction – Teachers must make certain they understand algebra well enough to teach it to their students. Effective teaching behaviors (e.g., specific praise, questioning) should also be included in all lessons.
- Error analysis – Error analysis is the process of looking closely at student errors to determine what they are doing incorrectly. Error analysis can be done by examining the problems or by interviewing students and asking them to demonstrate what they have done.
- Reviewing material – It is important that students receive ample opportunities to review what has previously been learned, in order for them to maintain the knowledge.
- Calculators – The use of calculators will assist students in completing complex math problems, including algebra. The use of calculators in algebra is complex, and students will need explicit instruction on how to use the calculators.
- Concrete materials – The use of concrete materials or manipulatives will assist students in understanding the abstract level of algebra. These manipulatives may include algebra tiles, algeblocks, or other items.
- Promoting a positive attitude toward math – Teachers must show enthusiasm when teaching algebra.

RESOURCES...

- Ashlock, R. A. (2006). *Error Patterns in Computation*. (9th ed.). Englewood Cliffs, NJ: Prentice Hall.
- Dixon, B. (1994). Research-based guidelines for selecting mathematics curriculum. *Effective School Practice*, Spring, 47–51.
- Salend, S. J., & Hofstetter, E. (1996). Adapting a problem-solving approach to teaching mathematics to students with mild disabilities. *Intervention in School and Clinic*, 31(4), 209–217.

BACKGROUND

Student: Sam

Age: 14.8

Grade: 9th

Focus: Basic operations in algebra

SCENARIO

It is the beginning of the school year and Sam's algebra class is reviewing the basic concepts of algebra. The concepts were briefly presented the previous year, and after a quick review most students are ready to move on to more advanced concepts. Sam, however, is having difficulty with the basic concepts. His teacher realizes these skills must be mastered before moving on to more advanced concepts, and she is willing to work with Sam to teach him strategies that will assist him in mastering the following goals:

- Solve addition, subtraction, multiplication, and division problems involving integers (e.g., $4 + 6$; $4 + -6$; 4×5 ; $-18 \div -3$).
- Simplify addition, subtraction, multiplication, and division equations (e.g., $(2x + 6) + (4x + 7) = 6x + 13$).
- Solve expressions with variables (e.g., $3x = -24$).
- Solve two-step equations (e.g., $3x - 4 = 2$).
- Solve multi-step equations (e.g., $5x - 4 = 2x + 5$).

POSSIBLE STRATEGIES

- Teaching Vocabulary
- Concrete-Representational-Abstract Method
- Graphic Organizers



ASSIGNMENT

1. Read General Guidelines for Teaching Algebra provided at the beginning of this case study.
2. Read the STAR Sheets for each possible strategy listed above.
3. Summarize the components of each strategy. Be sure to include how each strategy will support Sam and the benefits of using each strategy.

BACKGROUND

Student: Sheldon

Age: 14.6

Grade: 9th

Focus: Functions

SCENARIO

It is November and Sheldon's class has moved on to the algebra concepts of functions and graphs. Sheldon has been doing well in algebra class, receiving Bs the previous two grading periods. However, he is now beginning to have difficulty with the higher-level concepts of algebra. Sheldon has an excellent attitude and is willing to work with his teacher and a peer to meet the following goals:

- Determine whether a relation is a function, and describe the range of a function.
- Define and use the point-slope form.

POSSIBLE STRATEGIES

- Teaching Vocabulary
- Graphic Organizers



ASSIGNMENT

1. Read General Guidelines for Teaching Algebra provided at the beginning of this case study.
2. Read the STAR Sheets for each possible strategy listed above.
3. Summarize the components of each strategy. Be sure to include how each strategy will support Sheldon and the benefits of using each strategy.
4. Using one or more of these strategies, describe an activity that could be used to assist Sheldon.

BACKGROUND

Student: Tyisha

Age: 14.3

Grade: 9th

Focus: Solving real-world algebra problems

SCENARIO

Tyisha's teacher, Mr. Armstrong, puts a strong emphasis on relating algebra to real-world situations that affect his students' daily lives. When Tyisha is given problems to solve, she has no difficulty solving them. When given word problems requiring Tyisha to set up the problem before solving it, however, she has a great deal of difficulty. This is frustrating to Tyisha, as she knows how to perform the algorithm once the problem is written for her. Her teacher realizes several students are having difficulty with this concept and has decided to teach them a strategy to help with the following task:

- Write and solve the algebra equation in a real-life word problem.

POSSIBLE STRATEGIES

- Mnemonic Devices



ASSIGNMENT

1. Read the General Guidelines for Teaching Algebra provided at the beginning of this case study.
2. Read the STAR Sheet for the strategy listed above.
3. Summarize the components of the strategy. Be sure to include how the strategy will support Tyisha and the benefits of using the strategy.

BACKGROUND

Student: Maria

Age: 14.5

Grade: 9th

SCENARIO

It is the beginning of the second semester, and Maria is having a great deal of difficulty in her algebra class. She has an understanding of the basic concepts of algebra but has not mastered the skills needed to move to the higher-level concepts on which her class is working. Maria's teacher has spoken with her parents about the possible need for additional support, and her parents have agreed to help at home. They have identified the following goals for Maria:

- Solve addition, subtraction, multiplication, and division problems involving integers (e.g., $4 + 6$; $4 + -6$; 4×5 ; $-18 \div -3$).
- Simplify addition, subtraction, multiplication, and division equations (e.g., $(2x + 6) + (4x + 7) = 6x + 13$).
- Solve expressions with variables (e.g., $3x = -24$).
- Solve two-step equations (e.g., $3x - 4 = 2$).
- Solve multi-step equations (e.g., $5x - 4 = 2x + 5$).
- Write and solve the algebra equation in a real-life word problem.

POSSIBLE STRATEGIES

- Teaching Vocabulary
- Concrete-Representational-Abstract Method
- Graphic Organizers
- Mnemonic Devices



ASSIGNMENT

1. Read the STAR Sheets for the four strategies listed above.
2. Sequence Maria's goals in the order you would address them.
3. For each goal, identify a strategy and explain how and why it will assist Maria in reaching her goals.

BACKGROUND

Student: José

Age: 13.9

Grade: 9th

SCENARIO

José has been doing well in algebra. He understands the basic concepts of algebra and enjoys solving the problems. However, José is having difficulty with the concepts requiring higher-level reasoning skills. Specific areas of difficulty include determining functions and solving word problems. José has a positive attitude and is motivated to learn strategies that will assist him in passing his algebra course. His teacher has identified the following goals for José:

- Determine whether a relation is a function, and describe the range of a function.
- Define and use the point-slope form.
- Write and solve algebra equations for real-life word problems.

POSSIBLE STRATEGIES

- Teaching Vocabulary
- Graphic Organizers
- Mnemonic Devices



ASSIGNMENT

1. Read the STAR Sheets for the three strategies listed above.
2. Explain how and why each strategy could assist José in reaching his goals.
3. Explain how you would involve José's parents, and develop an activity from one of the strategies that José's parents could use at home.

OVERVIEW OF BASIC ALGEBRA SKILLS

- ✓ Add, subtract, multiply, and divide integers
- ✓ Add, subtract, multiply, and divide algebraic expressions
- ✓ Solve expressions with variables
- ✓ Solve two-step equations
- ✓ Solve multi-step equations
- ✓ Solve real-world algebra problems
- ✓ Understand the algebraic order of operations
- ✓ Graph coordinates
- ✓ Understand functions

BACKGROUND

Student: Martha

Age: 14.3

Grade: 9th

SCENARIO

Martha is a polite student who has a good attitude toward school and has good attendance. She also enjoys working in groups with her peers, and they enjoy working with her. Her teacher reports she is having some difficulty with the algebra concepts presented so far. The teacher states that Martha tries hard and has a basic understanding of what to do but has trouble answering the problems on paper when it is time to work by herself. It is the middle of the school year, and Martha's teacher is concerned that Martha will have difficulty with the higher-level concepts, as she is not at a level of proficiency in the basic skills.

AREAS OF STRENGTH

- Is proficient in basic facts
- Understands how to manipulate integers
- Can combine like terms



ASSIGNMENT

1. Develop three to four goals for Martha.
2. Using the Algebra STAR Sheets, select a strategy for each goal, and explain the benefit of using the strategy to address the corresponding goal.
3. Select one goal and describe an independent practice activity that will assist Martha in achieving that goal.

★ WHAT A STAR SHEET IS...

A STAR (Strategies And Resources) Sheet provides you with descriptions of strategies that will help you solve the case studies in this unit.

WHAT IT IS...

Explicit instruction in teaching new vocabulary terms requires the teacher to directly teach the pronunciation and definitions of new vocabulary words in a highly organized manner. In the area of algebra, it also requires teachers to provide students with examples of problem types.

WHAT THE RESEARCH AND RESOURCES SAY...

- The ability to solve word problems is dependent on understanding the language in such problems.
- To avoid memory gaps and misunderstanding, vocabulary should be explicitly pre-taught and reviewed before each new math lesson.
- Students often require examples illustrating the context of vocabulary words, and they should be encouraged to use the words in journals, presentations, and explanations of work.
- When learning is difficult and novel, teachers must provide support for their students.
- Teachers must model what they want students to learn and provide guided instruction, independent practice, and frequent feedback.
- Students must be given ample opportunities to practice the task in order to generalize the strategy for other settings.

TYPES OF ACTIVITIES TO IMPLEMENT...

- **Pre-teaching Vocabulary**
One easy way to increase students' vocabulary is to pre-teach vocabulary terms before students encounter the words in their math textbooks or during lectures. When pre-teaching vocabulary, it is important to teach a word within its context. Carnine, Silbert, & Kameenui (1997) suggest the following method for teaching new vocabulary words:
 - State the definitions, and have students repeat the definitions.
 - Provide students with good and bad examples of the words.
 - Review the new words along with previously learned words to ensure students have the words in their long-term memories.
- **Self-Correcting Activities**
Once students have explicitly been taught the new vocabulary terms, they can continue practicing the words by using self-correcting word cards. Students are given cards with the vocabulary words written on them and another set of cards with the definitions written on them. Students then match the word with the correct definition. As seen in the example on the next page, students can correct their work by making sure the word and the definition have the same symbol in the upper-right-hand corner. This activity can be completed with a peer, in small groups, or independently.

<p style="text-align: right;">✓</p> <p style="text-align: center;">Variable</p>	<p style="text-align: right;">✓</p> <p>A letter used to represent one or more numbers in an expression, equation, or inequality</p> <p>Example: $4b = 12$</p>
<p style="text-align: right;">★</p> <p style="text-align: center;">Integers</p>	<p style="text-align: right;">★</p> <p>The set of whole numbers and their opposites.</p> <p>Examples: 1 and -1; 20 and -20</p>
<p style="text-align: right;">♥</p> <p style="text-align: center;">Solution to an Equation</p>	<p style="text-align: right;">♥</p> <p>A set of values for the variables in an equation that make a true statement when substituted into the equation.</p> <p>Example: $4b = 12$. $4(3) = 12$ Substitute $b = 3$ $12 = 12$ True statement</p>

KEEP IN MIND...

- Students must be taught the vocabulary terms before working on the self-correcting and picture-card activities.
- Pre-teaching vocabulary and self-correcting activities can be used in peer-tutoring settings, small-group settings, or as independent work.
- It is important to provide an example of the definition to which students can relate.
- Although these activities may seem juvenile, students will appreciate having the vocabulary cards for reference as needed.

A list of important vocabulary terms for Case Studies A1 and A2 are provided on the next two pages.

RESOURCES...

Carnine, D. W., Silbert, J., & Kameenui, E. J. (Eds.). (1997). *Direct instruction reading* (3rd ed.). Upper Saddle River, NJ: Prentice Hall.

Center for the Education and Study of Diverse Populations. Connections in reading and mathematics instruction. (n.d.) Retrieved October 18, 2005, from <http://cesdp.nmhu.edu/pubs/connections.pdf>

Chard, D. Vocabulary strategies for the mathematics classroom. (2003). Houghton Mifflin Math [Online]. Retrieved October 18, 2005, from http://www.eduplace.com/state/pdf/author/chard_hmm05.pdf

Mercer, C. D., Lane, H. B., Jordan, L., Allsopp, D. H., & Eisele, M. R. (1996). Empowering teachers and students with instructional choices in inclusive settings. *Remedial and Special Education*, 17, 226–236.

Vaidva, S. R. (2004). Understanding dyscalculia for teaching. *Education*, (124)4, 717.

VOCABULARY TERMS

Here is a list of vocabulary terms appropriate for Level A - Case 1.

- Absolute Value: the distance from a point on the number line to zero
- Algebraic Equation: an equation that contains one or more variables (e.g., $x + 3 = 7$)
- Algebraic Expression: an expression that is written using one or more variables (e.g., $2x + 5$)
- Coordinate Plane: a plane that is divided into four regions by a horizontal and vertical number line
- Equation: a mathematical sentence that uses an equals sign to show that two quantities are equal
- Equivalent: having the same value
- Formula: a rule that is expressed with symbols
- Inclusion Symbols: parentheses $()$, brackets $\{\}$, and braces $[\]$ used in algebra problems
- Integers: the set of whole numbers and their opposites (e.g., 1 and -1)
- Like Terms: expressions that have the same variables and same powers of the variables
- Linear Equation: an equation whose graph is a straight line
- Numerical Expression: an expression that includes numbers and at least one operation (addition, subtraction, multiplication, or division)
- Order of Operations: the correct order in which the operations are performed within an expression
- Quadrant: one of the four regions into which a plane has been divided
- Rational Number: any number that can be expressed as a ratio a/b , where a and b are integers and $b \neq 0$
- Simplest Form: the form of an expression in which all like terms are combined
- Solution to an Equation: a set of values for the variables in an equation that makes a true statement when substituted into the equation
- Variable: a letter used to represent one or more numbers in an expression, equation, or inequality

VOCABULARY TERMS

Here is a list of vocabulary terms appropriate for Level A – Case 2.

- **Coordinate Plane:** a plane divided into four regions by a horizontal and vertical number line
- **Coordinates:** an ordered pair of numbers that locates a point in the coordinate plane in relation to the x- and y-axes
- **Dependent Variable:** the variable in a function whose value depends on the value of the other variable
- **Domain:** the first coordinates in a set of ordered pairs of a relation or function
- **Function:** a relation in which no two ordered pairs have the same x-value
- **Function Notation:** a method of writing a function in which the dependent variable is written in the form $f(x)$ — the independent variable, x , is placed in the parentheses
- **Function Table:** a table of ordered pairs that represent solutions of a function.
- **Independent Variable:** the variable in a function whose value does not depend on the value of the other variable
- **Negative Correlation:** a relationship between two variables in a scatter plot in which one variable increases while the other decreases
- **Origin:** the point of intersection of the x- and y-axes in the coordinate plane
- **Positive Correlation:** a relationship between two variables in a scatter plot in which both variables increase
- **Quadrant:** one of the four regions into which a plane has been divided
- **Range:** the second coordinates in a set of ordered pairs of a relation or function
- **Relation:** a pairing between two sets of numbers
- **Slope:** the measure of the steepness of a line; the ratio of vertical change to horizontal change
- **x-axis:** the horizontal number line on a coordinate plane
- **y-axis:** the vertical number line on a coordinate plane

★ WHAT A STAR SHEET IS...

A STAR (Strategies And Resources) Sheet provides you with descriptions of researched-based strategies that will help you solve the case studies in this unit.

WHAT IT IS...

The Concrete-Representational-Abstract (CRA) Method of teaching mathematical concepts is a method that allows students to understand a concept before memorizing the algorithms: 1) During the concrete stage, students interact and manipulate three-dimensional objects (e.g., algebra tiles, algebra manipulatives with representations of variables and units). This interaction will assist students in understanding the concept, instead of simply having them solve the algorithm. 2) During the representational stage, students use two-dimensional objects (e.g., pictures) to represent the problems. These pictures may be presented to them by the teacher or through the curriculum used in the class, or students may draw their own representations of the problem. 3) The abstract stage requires students to complete the algorithm without any concrete or representational assistance.

WHAT THE RESEARCH AND RESOURCES SAY...

- CRA is effective at all age levels and can assist students in learning basic concepts, operations, and applications.
- Students do not need a large amount of formal experience at the concrete and representational levels to understand the algorithms.
- Students demonstrate a conceptual understanding of the process when using this method, rather than just completing the algorithm.

TIPS FOR IMPLEMENTATION...

- Teachers must be very familiar with the concrete objects prior to teaching and having students interact with them.
- Teachers must provide modeling at all three stages of the CRA method.
- Teachers should continuously monitor student work during the concrete and representational levels, asking them questions about their thinking and providing clarification as needed.

TYPES OF ACTIVITIES TO IMPLEMENT...

- Algebra manipulatives:
 - Green squares represent 1. The squares have a “+” or a “-” on them.
 - Yellow rectangles represent x . The rectangles have a “+” or a “-” on them.
 - Other manipulatives can be used to show y , y^2 , y^3 , x^2 and x^3 .

ADDING AND SUBTRACTING NEGATIVE AND POSITIVE INTEGERS

1A.

$$4 + 6 = 10$$

Step 1: Write out the problem

$$4 + 6 =$$

Step 2: Using algebra manipulatives, show 4 positive squares and show 6 positive squares

Step 3: Add the squares together to get 10 positive squares

Step 4: Write out the answer

$$10$$

1B.

$$-4 + -6 = -10$$

Step 1: Write out the problem

$$-4 + -6 =$$

Step 2: Using algebra manipulatives, show 4 negative squares and show 6 negative squares

Step 3: Add the squares together to get 10 negative squares

Step 4: Write out the answer

$$-10$$

1C.

$$-4 + +6 = +2$$

Step 1: Write out the problem

$$-4 + +6 =$$

Step 2: Using algebra manipulatives, show 4 negative squares and show 6 positive squares

Step 3: Cross out equal pairs to get 2 positive squares

Step 4: Write out the answer

$$2$$

1D.

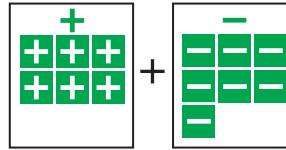
$$+6 + -7 = -1$$

Another method involves using a mat with a "+" sign on one side and a "-" sign on the other side.

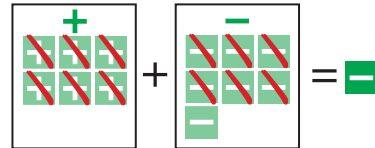
Step 1: Write out the problem

$$+6 + -7 =$$

Step 2: Using algebra manipulatives, show 6 blocks on the positive side and 7 blocks on the negative side



Step 3: Cross out equal pairs to get 1 negative square



Step 4: Write out the answer

$$-1$$

Please note: Before using algebra manipulatives, students should convert all subtraction problems to addition of negative numbers.

1E.



The use of a number line will assist students with moving in the correct direction. This process shows students how to start at -12 and then move in the direction of 8 positives, ending up at -4.

MULTIPLYING AND DIVIDING EXPRESSIONS

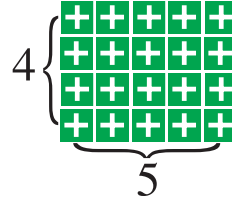
2A.

$$4 \cdot 5 = 20$$

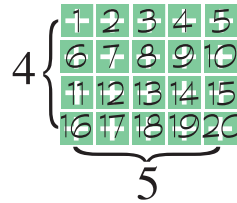
Step 1: Write out the problem

$$4 \cdot 5 =$$

Step 2: Using algebra manipulatives, show $4 \cdot 5$



Step 3: Students can count squares if needed, or they can count by 5's



Step 4: Write out the answer

20

(If the problem were $-4 \cdot 5$, you would use the same process but indicate the answer would be negative.)

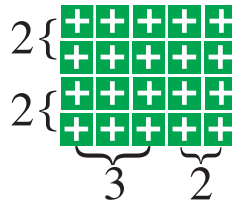
2B.

$$(2 + 2) \cdot (3 + 2) = 20$$

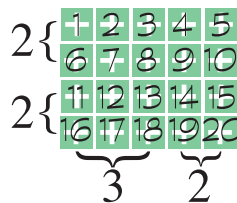
Step 1: Write out the problem

$$(2 + 2) \cdot (3 + 2) =$$

Step 2: Using algebra manipulatives, show $(2 + 2)$ AND $(3 + 2)$ Fill in tiles until you have a rectangle



Step 3: Students can count squares if needed, or they can count by 5's



Step 4: Write out the answer

20

2C.

$$\frac{(3x + 9)}{3} = x + 3$$

Step 1: Write out the problem

$$\frac{3x + 9}{3} =$$

Step 2: Using algebra manipulatives, show $\frac{(3x + 9)}{3} = x + 3$

Step 3: Students can cross off squares or stack them to better visualize the order of operations

Step 4: Divide the x's tiles and the 1's tiles by 3 respectively to show that the equation is equal to x + 3

Step 5: Write out the answer

$$x + 3$$

2D.

$$(2x + 6) + (4x + 7) = 6x + 13$$

Step 1: Write out the problem

$$(2x + 6) + (4x + 7) =$$

Step 2: Using algebra manipulatives, show $(2x + 6)$ AND $(4x + 13)$

Step 3: Combine the x's tiles and the 1's tiles

Step 4: Write out the answer

$$6x + 13$$

SOLVING EQUATIONS

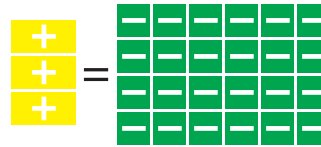
3A.

$$3x = -24 \text{ solve for } x$$

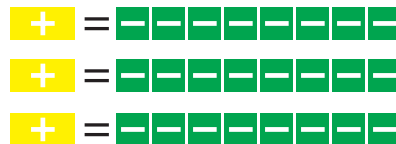
Step 1: Write out the equation

$$3x = -24$$

Step 2: Using algebra manipulatives, show $3x = -24$



Step 3: Separate each x tile, then divide the 1's tiles equally among the x's



Step 4: Write out the answer

$$x = -8$$

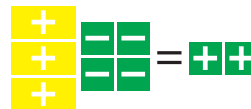
3B.

$$3x + -4 = 2 \text{ solve for } x$$

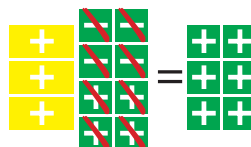
Step 1: Write out the equation

$$3x + -4 = 2$$

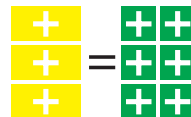
Step 2: Using algebra manipulatives, show $3x + -4 = 2$



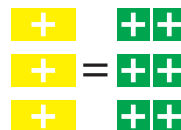
Step 3: Add positive 4 to each side of the equation, cross out negating pairs



Step 4: Simplify by removing the canceled tiles.



Step 5: Separate each x tile, then divide the 1's tiles equally among the x's



Step 6: Write out the answer

$$x = 2$$

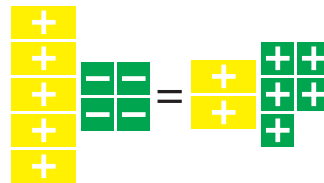
3C.

$$5x - 4 = 2x + 5 \text{ solve for } x$$

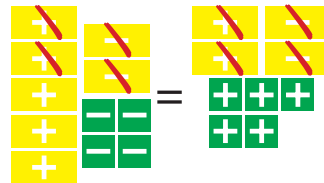
Step 1: Write out the equation

$$5x + 4 = 2x + 5$$

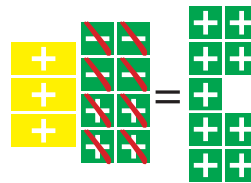
Step 2: Using algebra manipulatives, show $5x - 4 = 2x + 5$



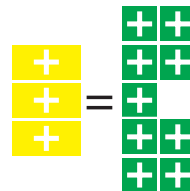
Step 3: Add negative 2x to both sides of the equation, cross out negating pairs



Step 4: Add positive 4 to each side of the equation, cross out the 4 negatives with the 4 positives



Step 5: Simplify by removing the canceled tiles.



Step 6: Separate each x tile, then divide the 1's tiles equally among the x's



Step 7: Write out the answer

$$x = 3$$



KEEP IN MIND...

- Activities during the concrete and representational stages must represent the actual process so that students are able to generalize the process during the abstract stage.
- Students must be able to manipulate the concrete objects; therefore, you must have enough objects for students to use either individually or in small groups (composed of no more than three students).

RESOURCES...

Gagnon, J. C., & Maccini, P. (2001). Preparing students with disabilities for algebra. *TEACHING Exceptional Children*, 34(1), 8–15.

Maccini, P., & Hughes, C. A. (1997). Mathematics interventions for adolescents with learning disabilities. *Learning Disabilities Research & Practice*, 12(3), 168–176.

Maccini, P., & Gagnon, J. C. (2000). Best practices for teaching mathematics to secondary students with special needs. *Focus on Exceptional Children*, 32(5), 1–22.

Witzel, B., Smith, S. W., & Brownell, M. T. (2001). How can I help students with learning disabilities in algebra? *Intervention in School and Clinic*, 37(2), 101–104.

Witzel, B. S., Mercer, C. D., & Miller, M. D. (2003). Teaching algebra to students with learning difficulties: An investigation of an explicit instruction model. *Learning Disabilities Research & Practice*, 18(2), 121–131.

★ WHAT A STAR SHEET IS...

A STAR (Strategies And Resources) Sheet provides you with descriptions of research-based strategies that will help you solve the case studies in this unit.

WHAT IT IS...

Graphic organizers are visuals that assist students in remembering information. In the area of algebra, graphic organizers may be used to provide students with formulas or cues needed to solve the problems.

WHAT THE RESEARCH AND RESOURCES SAY...

- Graphic organizers allow students to see relationships between ideas.
- Few language skills are necessary to understand graphic organizers.
- Teachers can use graphic organizers to diagnose areas of difficulty.
- The strategy can be generalized for use in many situations.
- Graphic organizers allow students to organize information.

TIPS FOR IMPLEMENTATION...

- Graphic organizers should be part of the instruction but not a substitute for it.
- Students must have an understanding of prerequisite skills needed to complete the graphic organizer.
- It is not necessary for students to memorize all procedures, but they need to know where to access the information (e.g., cue cards).
- Graphic organizers should not be complicated: the simpler, the better.

TYPES OF ACTIVITIES TO IMPLEMENT...

- Cue Cards can assist students in remembering the rules.

1	<p>Identify Property for Addition</p> <p>$a + 0 = a$ AND $0 + a = a$</p> <p>$7 + 0 = 7$ $0 + 7 = 7$</p>	2	<p>Addition Inverse Property</p> <p>$a + (-a) = 0$ AND $-a + a = 0$</p> <p>$5 + (-5) = 0$ $-5 + 5 = 0$</p>	3	<p>Definition of Subtraction</p> <p>$a - b = a + (-b)$</p>
4	<p>Adding Two Signed Numbers</p> <p>Addition of two numbers with like signs</p> <p>a. Find the sum of the numbers.</p> <p>b. Use the sign common to both numbers.</p> <p>Addition of two numbers with unlike signs</p> <p>a. Find the difference between the numbers</p> <p>b. Use the sign of the number with the greater absolute value.</p>	5	<p>Multiplying Two Signed Numbers</p> <p>$(+) \cdot (+) = (+)$ $(-) \cdot (-) = (+)$</p> <p>$5 \cdot 5 = 25$ $-5 \cdot -5 = 25$</p> <p>$(+) \cdot (-) = (-)$ $(-) \cdot (+) = (-)$</p> <p>$5 \cdot -5 = -25$ $-5 \cdot 5 = -25$</p>	6	<p>Dividing Two Signed Numbers</p> <p>$(+) \div (+) = (+)$ $(-) \div (-) = (+)$</p> <p>$36 \div 6 = 6$ $-36 \div -6 = 6$</p> <p>$(+) \div (-) = (-)$ $(-) \div (+) = (-)$</p> <p>$36 \div -6 = -6$ $-36 \div 6 = -6$</p>

- Tables also assist students in remembering information. For example, these two function tables provide the steps used to determine if there is a function. Students can keep these tables and steps in front of them as examples to follow while working.

Number of Days	Cost of Rentals (in dollars)	Steps:
1	15	1. Determine if this is a function. a. What are the ordered pairs? (1, 15); (2, 30); (3, 45); (4, 60); (5, 75); (6, 90); (7, 105); (8, 120); (9, 135); (10, 150) b. What is the domain? (1, 2, 3, 4, 5, 6, 7, 8, 9, 10) c. What is the range? (15, 30, 45, 60, 75, 90, 105, 120, 135, 150) d. Do any of the ordered pairs have the same first coordinate? No e. Is this a function? Yes
2	30	
3	45	
4	60	
5	75	
6	90	
7	105	
8	120	
9	135	
10	150	

Number of Apples	Cost of Apples (in dollars)	Steps:
1	1	1. Determine if this is a function. a. What are the ordered pairs? (1, 1); (2, 2); (4, 6); (2, 3); (5, 7); (3, 4); (2, 4); (5, 5); (4, 8); (1, 2) b. What is the domain? (1, 2, 4, 2, 5, 3, 2, 5, 4, 1) c. What is the range? (1, 2, 6, 3, 7, 4, 4, 5, 8, 2) d. Do any of the ordered pairs have the same first coordinate? Yes e. Is this a function? No
2	2	
4	6	
2	3	
5	7	
3	4	
2	4	
5	5	
4	8	
1	2	

RESOURCES...

Ives, B., & Hoy, C. (2003). Graphic organizers applied to higher-level secondary mathematics. *Learning Disabilities Research & Practice, 18*(1), 36–51.

Jitendra, A. (2002). Teaching students math problem-solving through graphic representations. *TEACHING Exceptional Children, 34*(4), 34–38.



WHAT A STAR SHEET IS...

A STAR (Strategies And Resources) Sheet provides you with descriptions of research-based strategies that will help you solve the case studies in this unit.

WHAT IT IS...

Mnemonic devices are words, sentences, or pictorial devices created to help students remember content.

WHAT THE RESEARCH AND RESOURCES SAY...

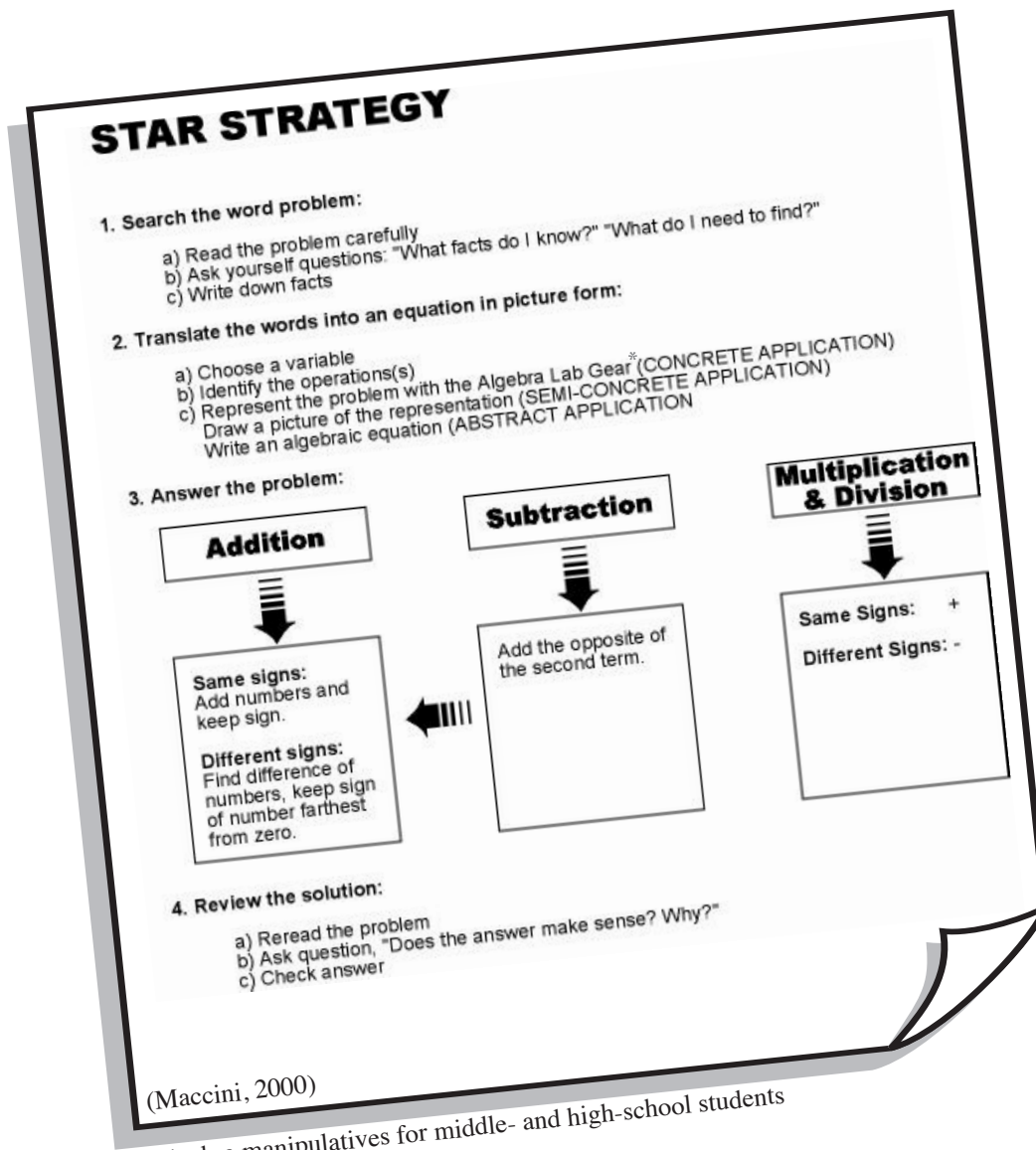
- Mnemonics can assist students in acquiring and retaining information.
- Mnemonics are effective for learning a new process.
- Mnemonics can be developed by either the teacher or the student.

TIPS FOR IMPLEMENTATION...

- Strategies must be modeled by the teacher prior to student use.
- Mnemonics must be appropriate and match the content being presented.
- The use of mnemonics must be explicitly taught.
- Ample practice is needed for students to independently use the mnemonics.

TYPES OF ACTIVITIES TO IMPLEMENT...

- STAR Strategy
STAR, a problem-solving strategy for algebra, stands for “Search, Translate, Answer, and Review.” Teachers may want to consider using a mnemonic device like STAR if students are having difficulty pulling key information out of word problems in order to formulate their equations. The figure on the following page illustrates the guidelines students should follow to apply STAR to algebra word problems. Students can use the method in combination with algebra manipulatives (concrete application), with pictorial representations (semi-concrete application), or with written algebraic equations (abstract application). Teachers need to ensure that students have mastered the concrete application before allowing them to move on to the semi-concrete or abstract phases.



* Algebra manipulatives for middle- and high-school students

RESOURCES...

- Maccini, P., & Hughes, C. A. (2000). Effects of a problem-solving strategy on the introductory algebra performance of secondary students with learning disabilities. *Learning Disabilities Research & Practice, 15*(1), 10–21.
- Prater, M. A. (1993). Teaching concepts: Procedures for the design and delivery of instruction. *Remedial and Special Education, 14*(5), 51–62.